Electric Field Vector in Terms of Scalar and Vector Potential

In classical electromagnetism, the electric field **E** is a fundamental vector quantity that describes the force experienced by a charged particle in the presence of electric and magnetic fields. The electric field can be expressed in terms of two auxiliary quantities: the **scalar potential** (**V**) and the **vector potential** (**A**). These potentials provide a more convenient way to analyze and solve problems involving electromagnetic fields, especially in dynamic systems.

Scalar Potential (V) and Vector Potential (A)

1. **Scalar Potential (V)**:
   - The scalar potential is a scalar field that represents the electric potential energy per unit charge.
   - It is primarily used in static (time-independent) electric fields and relates directly to the conservative nature of the electric field in such cases.
2. **Vector Potential (A):**
   - The vector potential is a vector field that is primarily used to describe the magnetic field in dynamic (time-dependent) systems.
   - It is linked to the magnetic field **B** through the relation: $\mathbf{B} = \nabla \times \mathbf{A}$.

Together, **V** and **A** allow the representation of both electric and magnetic fields in a unified framework, which is particularly useful in electrodynamics.

Electric Field in Terms of Potentials

The electric field **E** is related to the scalar potential **V** and the vector potential **A** through the following equation:

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

Explanation of the Terms:

1. $-\nabla \mathbf{V}$:
   - This term represents the component of the electric field associated with the scalar potential.
   - It corresponds to the electrostatic field, which arises from static charges.
   - In the absence of time-dependent effects, E=−∇V\mathbf{E} = -\nabla V.
2. $-\frac{\partial \mathbf{A}}{\partial t}$:
   - This term represents the time-dependent component of the electric field.
   - It arises due to the changing magnetic field, as described by Faraday's law of electromagnetic induction.

The combination of these two terms provides a complete description of the electric field, accounting for both static and dynamic contributions.